ASSIGNMENT # 3 ANSWERS

Q2. a) $I = I_0 \exp\left(-\frac{t}{RC}\right)$

$R = 50\,\Omega$

$C = 20\,\frac{\mu F}{cm^2} \times 0.25\,cm^2$

$= 5\,\mu F$

$\therefore RC = 50\,\Omega \times 5\,\mu F = 250\,\mu s$

find t such that $I = I_0/100$

$\frac{I_0}{100} = I_0 \exp\left(\frac{-t}{RC}\right)$

$\ln 100 = \frac{t}{RC}$

$t = RC \ln 100$

$\boxed{\cong 1.15\,ms}$

b) $C = \frac{\Delta Q}{\Delta E}$ $\Delta E = 0.25V - (-0.75V)$
$= 1V$

$C = 5\mu F$

$\Delta Q = C\Delta E$
$= 5\mu F \times 1V$

$\boxed{\Delta Q = 5\mu C}$

c) When integrating data numerically one should use the trapezoidal rule.

$$\text{Area} = \sum_i \left\{ (x_{i+1} - x_i) + \left[\tfrac{1}{2}(y_{i+1} - y_i)(x_{i+1} - x_i) \right] \right\}$$

Most spreadsheet programs use the trapezoidal rule in their integration routine. Excel, however, doesn't ~~use~~ have an integration function so many of you evaluated the area under the curve as

$$\text{Area} = \sum_j i_j \Delta t$$

This approximation is really only accurate for very small values of Δt.

I didn't deduct marks for this but for your interest I include these comments.

Using the trapezoidal rule ....

Sampling freq.	Interval time	Area (μc)	% error
200 kHz	5 μs	4.901	2%
100 KHz	10 μs	4.804	3.9%
20 KHz	50 μs	4.107	17.9%

$$\% \text{ error} = \left| \frac{\text{abs. value} - \text{actual value}}{\text{actual value}} \right| \times 100\%$$

ex. 5μs $\left| \frac{4.901 - 5.0}{5.0} \right| \times 100\% = 2\%$

d) If the electrode area is 100 times smaller than RC becomes 100 times smaller (RC now equals 2.5 μs)

The transient described above, but now with the much smaller area decays to 10% of I_o in only ~ 6 μs.

Measuring a transient with such a rapid decay would require extremely rapid sampling.

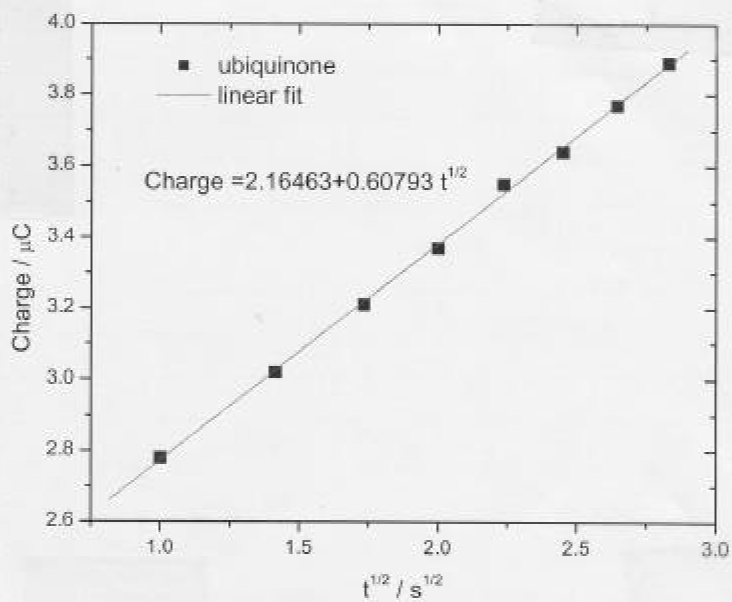
Even sampling at 1MHz (about the absolute limit of state of the art data acquisition systems) the data density would be too low to numerically evaluate the charge with an acceptable level of accuracy.

3. For a single-step, diffusion controlled transient

$$Q_t = 2nFA\,C_o^{bulk}\left(\frac{D_o t}{\pi}\right)^{1/2} + nFA\Gamma_o + Q_{DL}$$

By plotting Q_t vs $t^{1/2}$ the intercept of the best fit line is equal to

$$Q_{ads} + Q_{DL} = nFA\Gamma_o + Q_{DL}$$



$\therefore$ Ignoring Q_{DL} $nFA\Gamma_0 = 2.165\mu C$

$$\Gamma_0 = \frac{2.165\ \mu C}{2(96{,}485\ C/mol)(0.1\ cm^2)}$$

$$\boxed{\Gamma_0 = 1.12 \times 10^{-10}\ moles/cm^2}$$

b) $C = 5\frac{\mu F}{cm^2} \times 0.1\ cm^2 = 0.5\ \mu F$

$\Delta Q_{DL} = 0.5\mu F \times \Delta E$ where $\Delta E = (-0.45 + 0.10)V$
$= 0.35V$

$\Delta Q_{DL} = 0.5\mu F \times 0.35V$
$= 0.175\mu C$

$\therefore$ $Q_{ads} = 2.165\mu C - 0.175\mu C$
$= 1.99\ \mu C$

$$\boxed{\text{now } \Gamma_0 = 1.03 \times 10^{-10}\ moles\ cm^{-2}}$$

% error = 8.7%