

Problem Set #1
Fall 2006
SOLUTIONS

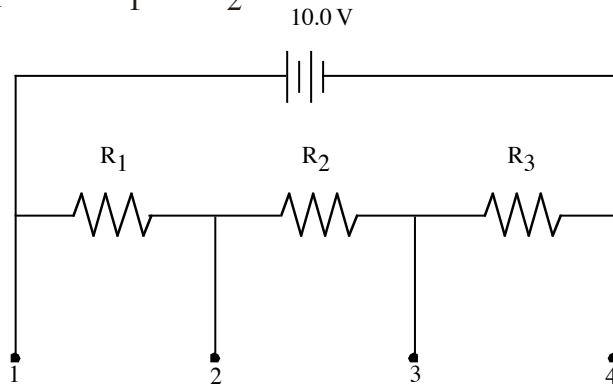
2.1(a) You can see that the voltage that is to drop across R_1 and R_2 is $1/2$ of the total voltage. Clearly the voltage drop across R_3 must be 5.0 V ; you are applying 10.0 V and 5.0 V is showing up across R_1 and R_2 so the rest must be across R_3 . In thinking about the voltage divider equation, this means that R_3 must be $1/2$ of the total resistance of the chain, or that

$$R_1 + R_2 = R_3$$

Also, from the voltage divider equation, we know that

$$\frac{R_1}{R_{total}} = \frac{1}{10} \quad \frac{R_2}{R_{total}} = \frac{4}{10}$$

$$\frac{R_1}{R_2} = \frac{1}{4}$$



Given that we only have two resistors of 50 W , 100 W and 200 W value, the only two possible conditions to meet this ratio is for $R_1 = 50\text{ W}$ and $R_2 = 200\text{ W}$ or else for $R_1 = 100\text{ W}$ and $R_2 = 400\text{ W}$ ($2 \times 200\text{ W}$). But since we know that R_3 must equal the sum of the other two, and since there are no 200 resistors leftover in the second case, it must be the first case that applies. Then $R_3 = 200\text{ W} + 50\text{ W} = 250\text{ W}$. Hence, we use 1 resistor for R_1 and R_2 but we must use 2 resistors in series to make up R_3 . To sum up

$$R_1 = 50\text{ W} \quad R_2 = 200\text{ W} \quad R_3 = 250\text{ W}$$

(b) As already reasoned, the IR (voltage) drop across R_3 must be 5.0 V .

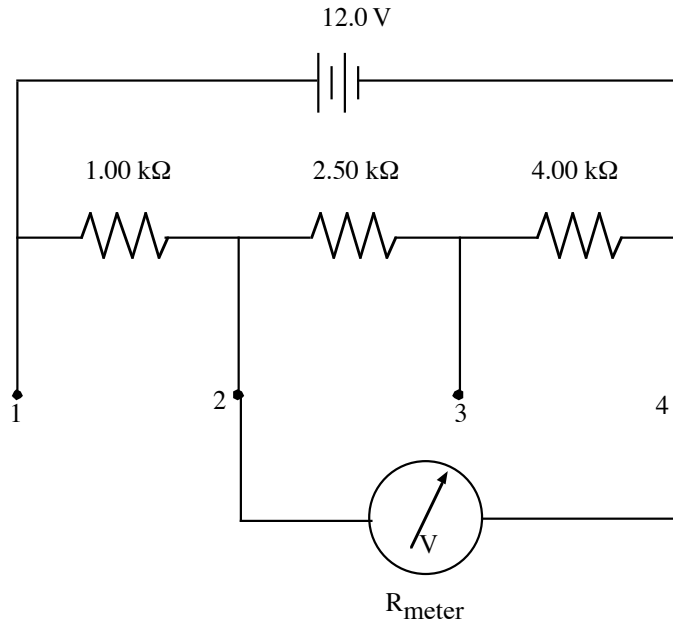
(c) Current is easily derived from the total resistance and total voltage

$$I = \frac{10.0\text{ V}}{(50 + 200 + 250)\text{ }\Omega} = \frac{10}{500} = 0.020\text{ A}$$

(d) Power is easily determined from the total voltage and current.

$$P = VI = 10.0\text{ V} \times 0.020\text{ A} = 0.20\text{ W}$$

2-3 When the meter is not connected, the voltage divider equation can be easily applied to determine the voltage drop between terminals 2 and 4. It is just the ratio of the resistance between the terminals being examined and the total resistance of the divider chain, times the total voltage.



$$V_{2,4} = 12.0 \left[\frac{2.5 + 4.0}{1.0 + 2.5 + 4.0} \right] = 12.0 \left[\frac{6.5}{7.5} \right] = 10.4 \text{ V}$$

When we put the meter in the circuit, the overall resistance changes because of this new path for current to flow. We can determine the effective resistance of this parallel portion of the network. The existing resistors behave as a single resistor with a resistance of $2.5 + 4.0 = 6.5 \text{ kW}$.

$$\frac{1}{R_{\text{eff}}} = \frac{1}{6.5} + \frac{1}{R_{\text{meter}}} = \frac{R_{\text{meter}} + 6.5}{R_{\text{meter}}(6.5)}$$

$$R_{\text{eff}} = \frac{R_{\text{meter}}(6.5)}{R_{\text{meter}} + 6.5}$$

Now substitute each meter resistance in turn, use the voltage divider equation to determine the voltage drop across R_{eff} and hence across the meter.

(a) $R_M = 5.00 \text{ kW}$

$$(a) R_{\text{eff}} = \frac{5.0(6.5)}{5.0 + 6.5} = 2.83 \text{ kW}$$

$$V_{\text{meter}} = 12.0 \left[\frac{2.83}{1.00 + 2.83} \right] = 8.87 \text{ V}$$

(b) $R_M = 50.0 \text{ kW}$

$$(b) R_{\text{eff}} = \frac{50.0(6.5)}{50.0 + 6.5} = 5.75 \text{ kW}$$

$$V_{\text{meter}} = 12.0 \left[\frac{5.75}{1.00 + 5.75} \right] = 10.22 \text{ V}$$

(c) $R_M = 500.0 \text{ k}\Omega$

$$(c) R_{eff} = \frac{500(6.5)}{500 + 6.5} = 6.42 \text{ k}\Omega$$

$$V_{\text{meter}} = 12.0 \left[\frac{6.42}{1.00 + 6.42} \right] = 10.38 \text{ V}$$

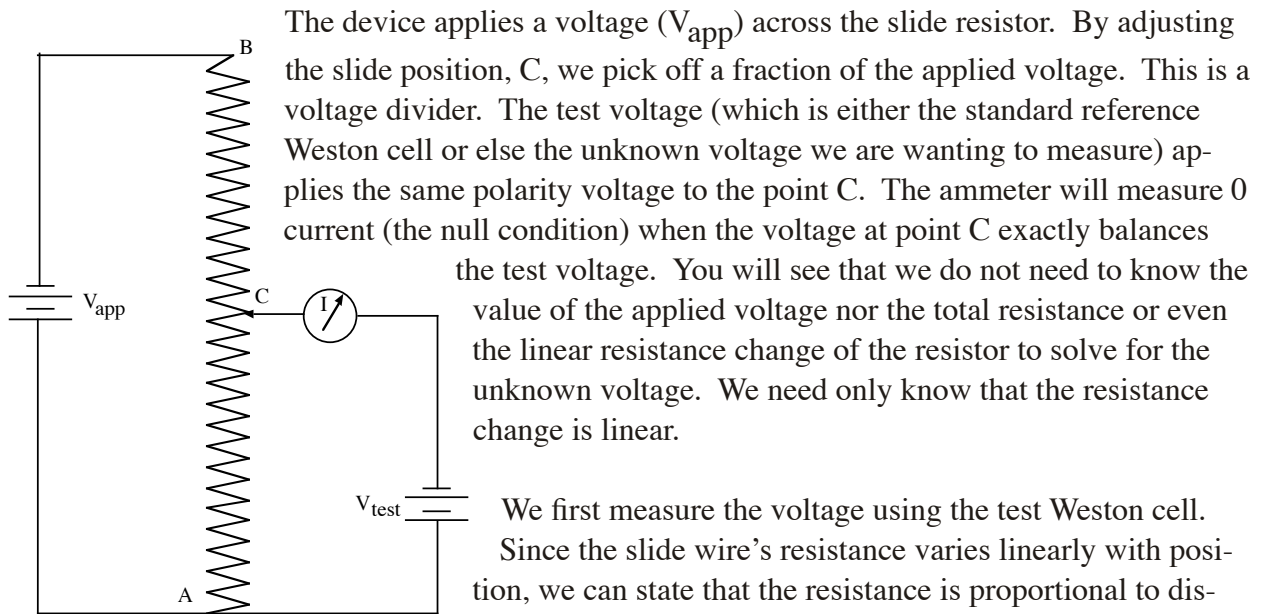
The relative error (report in percent) is found by taking the difference between the measured voltages and the voltage that should be present if the meter did not load the circuit.

$$(a) \% \text{Rel Error} = \frac{8.87 - 10.4}{10.4} \times 100\% = -15\%$$

$$(b) \% \text{Rel Error} = \frac{10.22 - 10.4}{10.4} \times 100\% = -1.7\%$$

$$(c) \% \text{Rel Error} = \frac{10.38 - 10.4}{10.4} \times 100\% = -0.19\%$$

2-7 The circuit as drawn in the textbook has additional complexity by offering the ability to switch the reference Weston cell or the unknown voltage source into the circuit. But to analyze the circuit, it will be a little clearer with the accompanying figure.



The device applies a voltage (V_{app}) across the slide resistor. By adjusting the slide position, C, we pick off a fraction of the applied voltage. This is a voltage divider. The test voltage (which is either the standard reference Weston cell or else the unknown voltage we are wanting to measure) applies the same polarity voltage to the point C. The ammeter will measure 0 current (the null condition) when the voltage at point C exactly balances the test voltage. You will see that we do not need to know the value of the applied voltage nor the total resistance or even the linear resistance change of the resistor to solve for the unknown voltage. We need only know that the resistance change is linear.

We first measure the voltage using the test Weston cell. Since the slide wire's resistance varies linearly with position, we can state that the resistance is proportional to distance along the wire.

$$V_{AC,1} = V_{app} \left[\frac{R_{AC}}{R_{AB}} \right] \quad \text{since } R \propto d$$

$$V_{AC,1} = V_{app} \left[\frac{d_{AC}}{d_{AB}} \right]$$

With the Weston cell, we know the voltage at C must equal 1.018 V and this occurs at the 84.3 cm position.

$$V_{AC,1} = V_{app} \left[\frac{84.3}{d_{AB}} \right] = 1.018 \text{ V}$$

$$\Rightarrow \frac{V_{app}}{d_{AB}} = \frac{1.018 \text{ V}}{84.3}$$

With the unknown, null occurs at 44.3 cm. With this and the knowledge of the ratio of the voltage to the resistor's length, we find the unknown voltage to be

$$V_{AC,2} = V_{app} \left[\frac{44.3}{d_{AB}} \right] = \left(\frac{V_{app}}{d_{AB}} \right) 44.3 = \left(\frac{1.018}{84.3} \right) 44.3 = 0.535 \text{ V}$$

2-14 and 2-15 These two problems are best solved together. We need to calculate the RC time constant for each circuit.

$$t_1 = R_1 C = 10 \text{ MW} \times 0.015 \text{ } \mu\text{F} = 0.15 \text{ s}$$

$$t_2 = R_2 C = 1 \text{ MW} \times 0.015 \text{ } \mu\text{F} = 15 \text{ ms}$$

$$t_3 = R_3 C = 1 \text{ kW} \times 0.015 \text{ } \mu\text{F} = 15 \text{ } \mu\text{s}$$

The discharge equation for a capacitor is simply that of an exponential decay. When the remaining voltage is 1% of the original voltage, we know that the ratio of $v(t)/V = 0.01$. We can solve for an expression to give us the time for different values of the time constant.

$$v(t) = V_{in} e^{-t/\tau}$$

$$\frac{v(t)}{V_{in}} = 0.01 = e^{-t/\tau}$$

$$\ln(0.01) = -\frac{t}{\tau}$$

$$t = -\tau \ln(0.01) = \tau \ln(100) = 4.605\tau$$

The three times are therefore

$$t_1 = 0.69 \text{ s}$$

$$t_2 = 69 \text{ m s}$$

$$t_3 = 69 \text{ } \mu\text{s}$$

2-18. The equations to find the properties of reactance, impedance and phase angle are just

$$X_c = \frac{1}{2\pi f C} \quad Z = \sqrt{R^2 + X_c^2} \quad \phi = -\arctan\left(\frac{X_c}{R}\right)$$

Here is a table showing all of the results.

Frequency	Resistance	Capacitance	Reactance	Impedance	Phase Angle
1	2×10^4	3.3×10^8	4.82×10^6	4.82×10^6	-89.8
1000	2×10^4	3.3×10^8	4.82×10^3	2.06×10^4	-13.6
1×10^6	2×10^4	3.3×10^8	4.82	2.00×10^4	-0.01
1	2×10^3	3.3×10^7	4.82×10^5	4.82×10^5	-89.8
1000	2×10^3	3.3×10^7	4.82×10^2	2.06×10^3	-13.6
1×10^6	2×10^3	3.3×10^7	0.482	2.00×10^3	-0.01
1	2×10^2	3.3×10^9	4.82×10^7	4.82×10^7	-90.0
1000	2×10^2	3.3×10^9	4.82×10^4	4.82×10^4	-89.8
1×10^6	2×10^2	3.3×10^9	48.2	2.06×10^2	-13.6

2-19 When you look at a low pass RC filter circuit, you realize that you can think of it like another voltage divider. However, the total resistance in this case is the network's impedance. For a high pass filter, we take off the voltage across the resistor, but for a low pass filter, we take off the voltage across the capacitor. The capacitor's reactance is like the portion of the resistance in the voltage divider network we are using, so the output voltage is just the ratio of the reactance to the impedance.

$$V_{p,out} = V_{p,in} \left[\frac{X_c}{Z} \right]$$

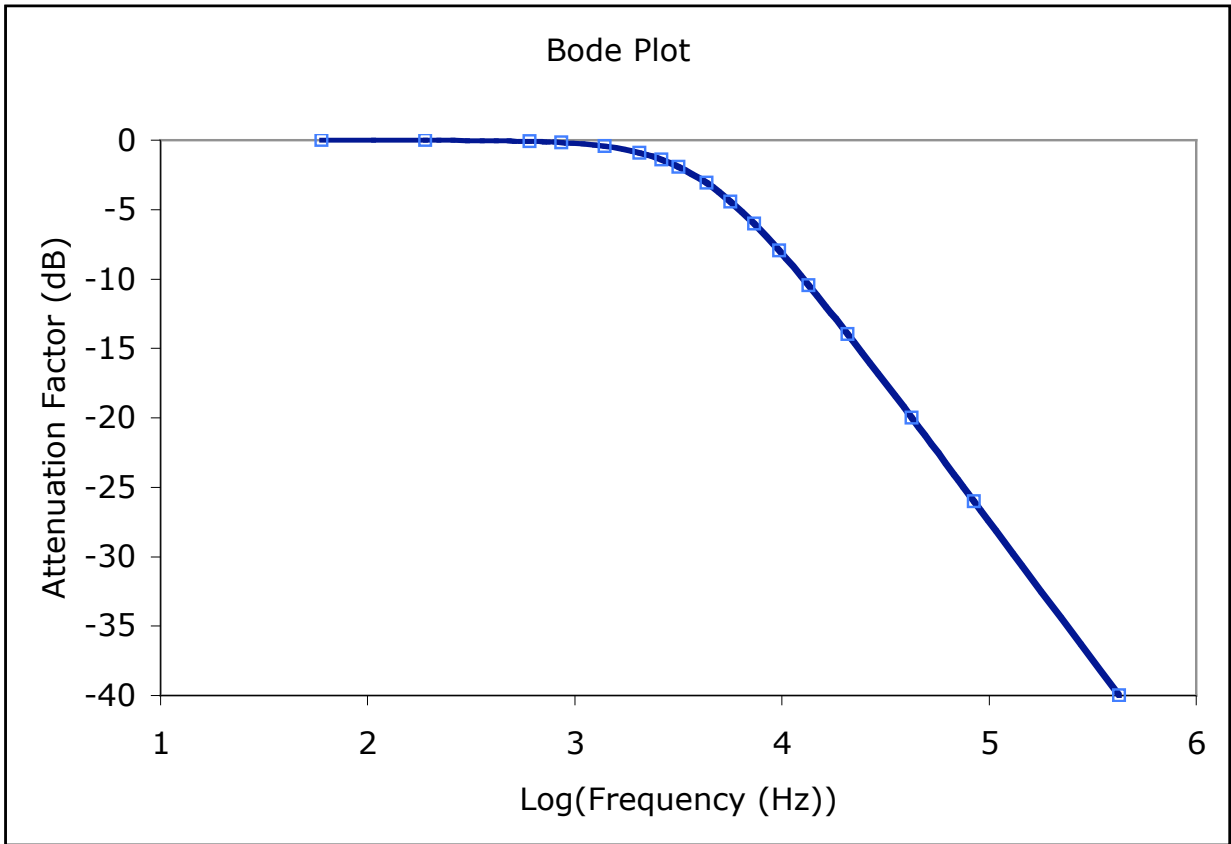
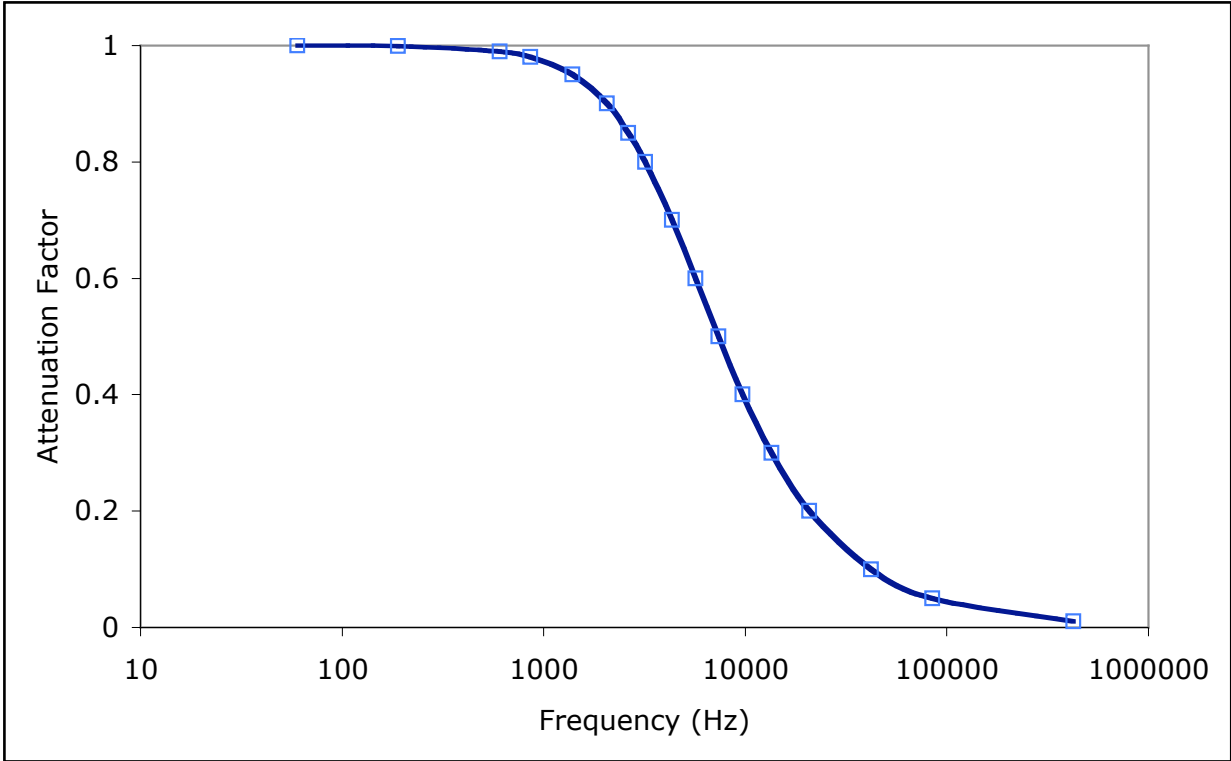
$$\frac{V_{p,out}}{V_{p,in}} = \frac{1}{2\pi f C} \times \frac{1}{\sqrt{R^2 + \left(\frac{1}{2\pi f C}\right)^2}}$$

$$\frac{V_{p,out}}{V_{p,in}} = \frac{1}{\sqrt{(2\pi f C R)^2 + 1}}$$

We can plot this graph for a range of frequencies, given the specified values for R and C. You can choose a range of frequencies by trial and error until you get the approximate values for the desired range for the voltage ratio. You might also consider rewriting the equation to solve for the frequency. I won't show the derivation here, but the result is

$$f = \frac{1}{2\pi C R} \sqrt{\left(\frac{V_{p,in}}{V_{p,out}}\right)^2 - 1}$$

Be careful to observe that the voltage ratio has flipped upside down in this expression. Note also that these equations are telling us about the ratio of the amplitude of the output voltage compared to the input voltage. If we plot 20 x log of this ratio and the log of the frequency, we will have a Bode plot, showing the attenuation of the signal in decibels. Here are both plots.



If you want the numbers, here they are

Voltage Attenuation	Frequency (Hz)
0.01	4.24×10^5
0.05	8.48×10^4
0.1	4.22×10^4
0.2	2.08×10^4
0.3	1.35×10^4
0.4	9.72×10^3
0.5	7.53×10^3
0.6	5.66×10^3
0.7	4.33×10^3
0.8	3.18×10^3
0.85	2.63×10^3
0.9	2.06×10^3
0.95	1.39×10^3
0.98	862
0.99	605
0.999	190
0.9999	60