These tables give the symmetry species of the product of two functions with the indicated symmetry species, for each point group. Although these are all immediately derivable from the character tables for each point group (for which, see any of the references listed in chapter 6), the direct product tables should suffice for most ordinary applications, such as determining selection rules, and so on. Also tabulated are the symmetry species of the components of the electromagnetic transition operators, as follows:

electric dipole: x, y, z;
magnetic dipole: R_x, R_y, R_z;
electric quadrupole: $x^2 + y^2, x^2 - y^2, z^2, xy, xz, yz$.

1.

C_1	A	Operators
A	A	All

2.

C_2	A	B	Operators
A	A	B	$z, R_z, x^2, y^2, z^2, xy$
B	B	A	x, y, R_x, R_y, xz, yz

3.

C_3	A	E	Operators
A	A	E	$z, R_z, x^2 + y^2, z^2$
E	E	$2A \oplus E$	$x, y, R_x, R_y, x^2 - y^2, xy, xz, yz$

4.

C_4	A	B	E	Operators
A	A	B	E	$z, R_z, x^2 + y^2, z^2$
B	B	A	E	$x^2 - y^2, xy$
E	E	E	$2A \oplus 2B$	x, y, R_x, R_y, xz, yz

5.

C_{2v}	A_1	A_2	B_1	B_2	Operators
A_1	A_1	A_2	B_1	B_2	z, x^2, y^2, z^2
A_2	A_2	A_1	B_2	B_1	R_z, xy
B_1	B_1	B_2	A_1	A_2	x, R_y, xz
B_2	B_2	B_1	A_2	A_1	y, R_x, yz

6.

C_{3v}	A_1	A_2	E	Operators
A_1	A_1	A_2	E	$z, x^2 + y^2, z^2$
A_2	A_2	A_1	E	R_z
E	E	E	$A_1 \oplus A_2 \oplus E$	$x, y, R_x, R_y, x^2 - y^2, xy, xz, yz$

7. C_{4v}: A_1, A_2, B_1, B_2, E

8. C_{5v}: A_1, A_2, E_1, E_2

9. C_{6v}: A_1, A_2, B_1, B_2, E_1, E_2

10. $C_s = C_{1h}$: A', A''

11. C_{2h}: A_g, A_u, B_g, B_u

12. C_{3h}: A', A'', E', E''

7.

C_{4v}	A_1	A_2	B_1	B_2	E	Operators
A_1	A_1	A_2	B_1	B_2	E	z, x^2+y^2, z^2
A_2	A_2	A_1	B_2	B_1	E	R_z
B_1	B_1	B_2	A_1	A_2	E	x^2-y^2
B_2	B_2	B_1	A_2	A_1	E	xy
E	E	E	E	E	$A_1 \oplus A_2 \oplus B_1 \oplus B_2$	x, y, R_x, R_y, xz, yz

8.

C_{5v}	A_1	A_2	E_1	E_2	Operators
A_1	A_1	A_2	E_1	E_2	z, x^2+y^2, z^2
A_2	A_2	A_1	E_1	E_2	R_z
E_1	E_1	E_1	$A_1 \oplus A_2 \oplus E_2$	$E_1 \oplus E_2$	x, y, R_x, R_y, xz, yz
E_2	E_2	E_2	$E_1 \oplus E_2$	$A_1 \oplus A_2 \oplus E_1$	x^2-y^2, xy

9.

C_{6v}	A_1	A_2	B_1	B_2	E_1	E_2	Operators
A_1	A_1	A_2	B_1	B_2	E_1	E_2	z, x^2+y^2, z^2
A_2	A_2	A_1	B_2	B_1	E_1	E_2	R_z
B_1	B_1	B_2	A_1	A_2	E_2	E_1	
B_2	B_2	B_1	A_2	A_1	E_2	E_1	
E_1	E_1	E_1	E_2	E_2	$A_1 \oplus A_2 \oplus E_2$	$B_1 \oplus B_2 \oplus E_1$	x, y, R_x, R_y, xz, yz
E_2	E_2	E_2	E_1	E_1	$B_1 \oplus B_2 \oplus E_1$	$A_1 \oplus A_2 \oplus E_2$	x^2-y^2, xy

10.

$C_s = C_{1h}$	A'	A''	Operators
A'	A'	A''	$x, y, R_z, x^2, y^2, z^2, xy$
A''	A''	A'	z, R_x, R_y, xz, yz

11.

C_{2h}	A_g	A_u	B_g	B_u	Operators
A_g	A_g	A_u	B_g	B_u	R_z, x^2, y^2, z^2, xy
A_u	A_u	A_g	B_u	B_g	z
B_g	B_g	B_u	A_g	A_u	R_x, R_y, xz, yz
B_u	B_u	B_g	A_u	A_g	x, y

12.

C_{3h}	A'	A''	E'	E''	Operators
A'	A'	A''	E'	E''	R_z, x^2+y^2, z^2
A''	A''	A'	E''	E'	z
E'	E'	E''	$2A' \oplus E'$	$2A'' \oplus E''$	x, y, x^2-y^2, xy
E''	E''	E'	$2A'' \oplus E''$	$2A' \oplus E'$	R_x, R_y, xz, yz

13.

C_{4h}	A_g	A_u	B_g	B_u	E_g	E_u	Operators
A_g	A_g	A_u	B_g	B_u	E_g	E_u	R_z, x^2+y^2, z^2
A_u	A_u	A_g	B_u	B_g	E_u	E_g	z
B_g	B_g	B_u	A_g	A_u	E_g	E_u	x^2-y^2, xy
B_u	B_u	B_g	A_u	A_g	E_u	E_g	
E_g	E_g	E_u	E_g	E_u	$2A_g \oplus 2B_g$	$2A_u \oplus 2B_u$	R_x, R_y, xz, yz
E_u	E_u	E_g	E_u	E_g	$2A_u \oplus 2B_u$	$2A_g \oplus 2B_g$	x, y

14.

D_2	A_1	B_1	B_2	B_3	Operators
A_1	A_1	B_1	B_2	B_3	x^2, y^2, z^2
B_1	B_1	A_1	B_3	B_2	z, R_z, xy
B_2	B_2	B_3	A_1	B_1	y, R_y, xz
B_3	B_3	B_2	B_1	A_1	x, R_x, yz

15.

D_3	A_1	A_2	E	Operators
A_1	A_1	A_2	E	x^2+y^2, z^2
A_2	A_2	A_1	E	z, R_z
E	E	E	$A_1 \oplus A_2 \oplus E$	$x, y, R_x, R_y, xz, yz, x^2-y^2, xy$

16.

D_4	A_1	A_2	B_1	B_2	E	Operators
A_1	A_1	A_2	B_1	B_2	E	x^2+y^2, z^2
A_2	A_2	A_1	B_2	B_1	E	z, R_z
B_1	B_1	B_2	A_1	A_2	E	x^2-y^2
B_2	B_2	B_1	A_2	A_1	E	xy
E	E	E	E	E	$A_1 \oplus A_2 \oplus B_1 \oplus B_2$	x, y, R_x, R_y, xz, yz

17.

D_5	A_1	A_2	E_1	E_2	Operators
A_1	A_1	A_2	E_1	E_2	x^2+y^2, z^2
A_2	A_2	A_1	E_1	E_2	z, R_z
E_1	E_1	E_1	$A_1 \oplus A_2 \oplus E_2$	$E_1 \oplus E_2$	x, y, R_x, R_y, xz, yz
E_2	E_2	E_2	$E_1 \oplus E_2$	$A_1 \oplus A_2 \oplus E_1$	x^2-y^2, xy

18.

D_6	A_1	A_2	B_1	B_2	E_1	E_2	Operators
A_1	A_1	A_2	B_1	B_2	E_1	E_2	x^2+y^2, z^2
A_2	A_2	A_1	B_2	B_1	E_1	E_2	z, R_z
B_1	B_1	B_2	A_1	A_2	E_2	E_1	
B_2	B_2	B_1	A_2	A_1	E_2	E_1	
E_1	E_1	E_1	E_2	E_1	$A_1 \oplus A_2 \oplus E_2$	$B_1 \oplus B_2 \oplus E_1$	x, y, R_x, R_y, xz, yz
E_2	E_2	E_2	E_2	E_1	$B_1 \oplus B_2 \oplus E_1$	$A_1 \oplus A_2 \oplus E_1$	x^2-y^2, xy

19.

D_{2d}	A_1	A_2	B_1	B_2	E	Operators
A_1	A_1	A_2	B_1	B_2	E	$x^2 + y^2, z^2$
A_2	A_2	A_1	B_2	B_1	E	R_z
B_1	B_1	B_2	A_1	A_2	E	$x^2 - y^2$
B_2	B_2	B_1	A_2	A_1	E	z, xy
E	E	E	E	E	$A_1 \oplus A_2 \oplus B_1 \oplus B_2$	x, y, R_x, R_y, xz, yz

20. D_{3d} has the same product rules and operator species as D_3 (table 15), with the addition of $g \otimes g = g$, $u \otimes u = g$, and $u \otimes g = u$. See table 11 for the g–u characters of the operators.

21. D_{2h} has the same product rules and operator species as D_2 (table 14), with the addition of $g \otimes g = g$, $u \otimes u = g$, and $u \otimes g = u$. See table 11 for the g–u characters of the operators.

22.

D_{3h}	A_1'	A_2'	A_1''	A_2''	E'	E''	Operators
A_1'	A_1'	A_2'	A_1''	A_2''	E'	E''	$x^2 + y^2, z^2$
A_2'	A_2'	A_1'	A_2''	A_1''	E'	E''	R_z
A_1''	A_1''	A_2''	A_1'	A_2'	E''	E'	
A_2''	A_2''	A_1''	A_2'	A_1'	E''	E'	z
E'	E'	E'	E''	E''	$A_1' \oplus A_2' \oplus E'$	$A_1'' \oplus A_2'' \oplus E''$	$x, y, x^2 - y^2, xy$
E''	E''	E''	E'	E'	$A_1'' \oplus A_2'' \oplus E''$	$A_1' \oplus A_2' \oplus E'$	R_x, R_y, xz, yz

23. D_{4h} has the same product rules and operator species as D_4 (table 16), with the addition of $u \otimes u = g$, $g \otimes g = g$, $u \otimes g = u$. See table 11 for the g–u character of the operator. D_{6h} is similarly derived from D_6 (table 18).

24.

O, T_d	A_1	A_2	E	F_1	F_2	Operators
A_1	A_1	A_2	E	F_1	F_2	$x^2 + y^2 + z^2$
A_2	A_2	A_1	E	F_2	F_1	
E	E	E	$\{A_1 \oplus A_2 \oplus E\}$	$F_1 \oplus F_2$	$F_1 \oplus F_2$	$2z^2 - x^2 - y^2, \sqrt{3}(x^2 - y^2)$
F_1	F_1	F_2	$F_1 \oplus F_2$	$A_1 \oplus E \oplus F_1 \oplus F_2$	$A_2 \oplus E \oplus F_1 \oplus F_2$	$\{R_x, R_y, R_z;\ x, y, z(O)\}$
F_2	F_2	F_1	$F_1 \oplus F_2$	$A_2 \oplus E \oplus F_1 \oplus F_2$	$A_1 \oplus E \oplus F_1 \oplus F_2$	$\{x, y, z(T_d);\ xz, xy, yz(O)\}$

25. O_h has the same product rules and operator species as O (table 24), with the addition of $g \otimes g = g$, $u \otimes u = g$, and $u \otimes g = u$. See table 11 for the u–g character of the operators.

26.

$C_{\infty v}$	Σ^+	Σ^-	Π	Δ	...	Operators
Σ^+	Σ^+	Σ^-	Π	Δ	...	$x^2 + y^2$, z^2, z
Σ^-	Σ^-	Σ^+	Π	Δ	...	R_z
Π	Π	Π	$\Sigma^+ \oplus \Sigma^- \oplus \Delta$	$\Pi \oplus \Phi$	...	x, y, R_x, R_y, xz, yz
Δ	Δ	Δ	$\Pi \oplus \Phi$	$\Sigma^+ \oplus \Sigma^- \oplus \Gamma$	...	$x^2 - y^2$, xy
⋮	⋮	⋮	⋮	⋮		

27.

$D_{\infty h}$	Σ_g^+	Σ_u^+	Σ_g^-	Σ_u^-	Π_g	Π_u
Σ_g^+	Σ_g^+	Σ_u^+	Σ_g^-	Σ_u^-	Π_g	Π_u
Σ_u^+	Σ_u^+	Σ_g^+	Σ_u^-	Σ_g^-	Π_u	Σ_g
Σ_g^-	Σ_g^-	Σ_u^-	Σ_g^+	Σ_u^+	Π_g	Π_u
Σ_u^-	Σ_u^-	3_g^-	Σ_u^+	Σ_g^+	Π_u	Π_g
Π_g	Π_g	Π_u	Π_g	Π_u	$\Sigma_g^+ \oplus \Sigma_g^- \oplus \Delta_g$	$\Sigma_u^+ \oplus \Sigma_u^- \oplus \Delta_u$
Π_u	Π_u	Π_g	Π_u	Π_g	$\Sigma_u^+ \oplus \Sigma_u^- \oplus \Delta_u$	$\Sigma_g^+ \oplus \Sigma_g^- \oplus \Delta_g$
Δ_g	Δ_g	Δ_u	Δ_g	Δ_u	$\Pi_g \oplus \Phi_g$	$\Pi_u \oplus \Phi_u$
Δ_u	Δ_u	Δ_g	Δ_u	Δ_g	$\Pi_u \oplus \Phi_u$	$\Pi_g \oplus \Phi_g$
⋮	⋮	⋮	⋮	⋮	⋮	⋮

$D_{\infty h}$	Δ_g	Δ_u	...	Operators
Σ_g^+	Δ_g	Δ_u	...	z^2, $x^2 + y^2$
Σ_u^+	Δ_u	Δ_g	...	z
Σ_g^-	Δ_g	Δ_u	...	R_z
Σ_u^-	Δ_u	Δ_g	...	
Π_g	$\Pi_g \oplus \Phi_g$	$\Pi_u \oplus \Phi_u$	...	R_x, R_y, xz, yz
Π_u	$\Pi_u \oplus \Phi_u$	$\Pi_g \oplus \Phi_g$	...	x, y
Δ_g	$\Sigma_g^+ \oplus \Sigma_g^- \oplus \Gamma_g$	$\Sigma_u^+ \oplus \Sigma_u^- \oplus \Gamma_u$	...	$x^2 - y^2$, xy
Δ_u	$\Sigma_u^+ \oplus \Sigma_u^- \oplus \Gamma_u$	$\Sigma_g^+ \oplus \Sigma_g^- \oplus \Gamma_g$	...	
⋮	⋮	⋮		

Appendix B

Newtonian m
many problem
Lagrange's eq
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$L \equiv T - V$,

a function

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) -$

where q_i a
will be sho

1 Some

We begin

$\mathbf{v} = (\dot{x}, \dot{y},$

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and x, y

1. This ap
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